Information Visualization in Mathematics - Abstract

Visualization in mathematics envelops mathematical thinking via visual representations, e.g. diagrams and computer images, and internal imagery about mathematics, e.g. imagining a cube and counting its vertices [9][10]. It has been, and still is, a predominant view in the epistemology of mathematics that visualization in mathematics cannot be a source of epistemic value. Some of the towering figures in mathematics in the past few centuries, such as Dedekind, Pasch, Hilbert, and Russell, expressed their scepticism towards the use of visual means in doing as well as thinking of mathematics. Nevertheless, a number of authors have recently defended the value of visualization. Some of them, such as Nelsen, Zimmerman, and Cunningham pointed to psychological, but others even to the epistemic value of visualizations, such as Azzouni, Giaquinto and Brown [2][9][10][11].

In this paper, I wish to join the latter group and defend the view that visualization really can be an epistemic artefact that has a role in justifying and even proving facts about mathematics. In doing so, I aim at showing two things. First, that one can acquire mathematical knowledge through a provided visualization of mathematical information. For example, I can acquire knowledge of the Pythagorean theorem by looking at a square positioned in a bigger square creating four identical triangles. Second, that one agent can transmit some mathematical information to another agent via visual means, as teachers and professors have done throughout our education. My claim is that it is important to develop such a position so that we can adequately account for a range of situations that we find ourselves in as epistemic agents. To contrast, if we accept the predominant view, by which mathematical knowledge is knowledge with a uniform witness - the deductive proof, we are unable to ascribe knowledge to agents or status of proof to demonstrations in situations when we clearly should; during lectures and oral exams in the former and journal articles in the latter case, for instance.

So, with this paper, I want to engage in the discussion within the epistemology of mathematics in the context of visual proof methods - such as

diagrams, as well as the philosophy of information in the context of information visualization. This paper rests primarily on the contributions from Bolinska [3][4], Giaquinto [9][10] and Azzouni [1][2] when it comes to showing the value of visual thinking, D'Agostino [5][6] in defending the notion of mathematical information and Löwe, Müller [11] and Floridi [7][8] with regards to their analysis of knowledge.

I will begin the investigation of the role of visualizations in mathematics by looking at the practice of mathematics first. Although the practice does not have to determine how we should define proof, justification, and knowledge, it definitely should be our starting point, since we should strive for describing actual epistemic situations with our models of knowledge [10][11]. So, for instance, we should not attribute knowledge by looking at how lecturers attribute knowledge to their students during exams. Nevertheless, the way we test mathematical knowledge on any educational level should be taken into consideration at the start of our examination. Furthermore, for this proposal to be adequate, it will be informed by cognitive science, semiotics, and philosophy of information, most notably in the contexts of visual experience and meaning comprehension [10]. Also, to be able to speak about the epistemic character of visualization of mathematical information, I will first show what do notions of mathematical information and mathematical knowledge mean.

Hence, the paper is divided into three parts. In the first part, the notion of mathematical information is investigated following the works of D'Agostino and Floridi. The main objective is to establish mathematical information as an object of mathematical knowledge. I will argue for acceptance of mathematical information on three grounds; coherence, explanatory power and semantic power. The first claim is that it is consistent to hold that there is such a thing as mathematical information and that its acceptance fits coherently into the web of informational concept we usually accept. The second claim is that the notion of information enables us to solve major paradoxes that are a conceptual burden to the orthodox view. The third claim is that we can use the concept of information to change our discourse about mathematics, which is heavily influenced by its platonic past, to a clearer and more appropriate alternative. Also, I will examine the features needed for a visualization to be adequate, where I am drawing on Azzouni, Giaquinto and, especially, Bolinska; namely, semantic and syntactic salience [4].

In the second part, I will set forth a definition of mathematical knowledge in informational terms that will characterize it as contextual and gradient. Here, I am drawing heavily on Löwe's and Müller's contextual definition of mathematical knowledge, as well as the network theory of account following

Floridi's proposal. By combining these two notions, I will be able to account for the epistemology of mathematical practice and it will enable me to show in what way can we think of mathematical visualizations as epistemic artefacts. That is, I will provide a formalized definition that is flexible enough for everyday use and evasion of traditional counterexamples, while being rigid enough for further philosophical considerations.

Finally, I will show how and when visualizations in mathematics can transmit information, as well as how and when an agent can acquire and/or demonstrate her mathematical knowledge through visualization of mathematical information. The epistemic character of visualizations is constrained both with the features needed for something to count as an adequate visualization and with the context that is embedded in our definition of mathematical knowledge. I will argue that if one accepts the approach taken in the first two parts of the paper, one is obliged to accept the desired epistemic character of mathematical visualizations. Furthermore, I will show cases, such as Venn diagrams, where visualization is the most natural and effective technique of such demonstrations.

Bibliography

1. Azzouni, J., (2005) Is there still a Sense in which Mathematics can have Foundations?, *Essays on the Foundations of Mathematics and Logic*, Giandomenico Sica (ed.), pp. 9-48.

2. Azzouni, J., (2013) That we see that some diagrammatic proofs are perfectly rigorous, *Philosophia Mathematica*, 21: pp. 323-338.

3. Bolinska, A. (2013) Epistemic representation, informativeness and the aim of faithful representation, *Synthese*, 190: 219-34.

4. Bolinska, A. (2016) Successful visual epistemic representation, *Studies in History and Philosophy of Science Part A*, 56: 153-160.

5. D'Agostino, M., (2016) The philosophy of mathematical information, *Routledge Handbook of Philosophy of Information*, Luciano Floridi (ed.), pp. 164-179.

6. D'Agostino, M., (2013) Semantic Information and the Trivialization of Logic: Floridi on the Scandal of Deduction, *Information* 4: pp. 33-59.

7. Floridi, L., (2011) The Philosophy of Information, Oxford, OUP

8. Floridi, L., (2012) Semantic information and the network theory of account, *Synthese*, 184: pp. 431-454

9. Giaquinto, M., (Winter 2016 Edition) The Epistemology of Visual Thinking in Mathematics, The Stanford Encyclopedia of Philosophy, Edward N. Zalta (ed.)

10. Giaquinto, M., (2007) Visual Thinking in Mathematics: An Epistemological Study, Oxford, OUP

11. Löwe, B., Müller, T., (2008) Mathematical Knowledge is Context Dependent, *Grazer Philosophische Studien*, 76: pp. 91-107.